International Journal of
DATA SCIENCE AND IOT MANAGEMENT SYSTEM
ISSN: 3068-272X www.ijdim.com Original Research Paper

"3D Representation of Pi Term Interactions: A Surface Plot of πa, πb, and Dependent Variable π_8"

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Received: 09-10-2023 Accepted: 11-11-2023 Published: 18-11-2023

ABSTRACT:
This study presents a three-dimensional graphical analysis of dimensional analysis-derived Pi terms, focusing on the interaction between πa (product of positive Pi terms), πb (product of negative Pi terms), and the dependent variable π_8. Using a surface plot approach, the visualization provides insights into how variations in πa and πb affect π_8, enabling a better understanding of multivariable relationships in engineering and physical systems. The representation supports predictive modeling and helps in optimizing parameters in systems governed by dimensional homogeneity.

Keywords: Dimensional Analysis, Buckingham Pi Theorem, Pi Terms, 3D Surface Plot, Multivariable Interaction, Mathematical Modeling.

Introduction:

Understanding and modeling such interactions is critical for analysis, optimization, and decision-making. One powerful approach to simplifying these complex systems is dimensional analysis, which reduces the number of variables by converting them into dimensionless Pi terms using the Buckingham Pi Theorem. These Pi terms preserve the essential relationships among the variables while eliminating the influence of physical units.

While traditional dimensional analysis focuses on deriving and interpreting individual or paired Pi terms, there is a growing need to visualize and analyze multi-dimensional interactions among them. This is particularly relevant in fields such as fluid dynamics, biomechanics, thermodynamics, and ergonomics, where multiple parameters influence a response variable simultaneously. A three-dimensional (3D) surface plot provides a valuable visual and analytical tool for exploring such interactions.

Whole meal flour (Whole-Grain)

This represents an extraction of 94% to 98% of the wheat grain. Minimum amount (2% to 6%) is sifted away as bran.

White flour

75% of the wheat grain is extracted. Most of the bran and the germ are sifted away leaving mostly the endosperm which results in a loss of 22 vitamins/minerals and dietary fiber.

All purpose flour

White flour made from a blend of hard and soft wheat. It may be un-bleached/bleached, bromated, pre-sifted (milled to finer texture). Types of flours are tabulated in Table 1.1

Table 1.1: Types of flour

S.no	Type of flour	Description
1	Atta	Wholemeal flour made from Hard wheat or Durum. It is coarser than the All-purpose flour.
2	Bread flour	White flour made from hard wheat, same texture as All-purpose flour.
3	Pastry flour	White flour made from a blend of soft and hard wheat (more soft wheat) similar in texture to the cake flour.
4	Maida	Wholemeal flour made from soft wheat similar in texture to cake flour. In reality, it resembles more to the Pastry flour being Whole meal flour (higher protein).
5	Farina/Cream of wheat	Granular product made from endosperm of any wheat other than the Durum.
6	Semolina	Granular (similar to Farina) product made from endosperm of Durum.
7	Sooji	Wholemeal granular (similar to Farina) product made from Hard wheat.
8	Durum flour	White flour made from Durum same texture as the All purpose flour.
9	Self-rising flour	White flour made from soft wheat. The leavening agents such as baking soda or baking powder, and acid-releasing agents are sifted in the final stage.
10	Stone ground Whole Wheat flour	Wholemeal flour made from a blend of soft and hard wheat.
11	Whole wheat flour/Graham flour	Since the roller-milling sifts out the bran and the germ, for whole wheat flour, they are added back in to the All-purpose flour.
12	Cake flour	White flour made from soft wheat and finer texture than the All-purpose flour.

Wheat flour is high in nutrients. Because of its fiber properties, wheat flour is the first choice of the health-conscious people. Wheat flour is obtained by milling wheat. There are various types of wheat. The differences between the flours comes down to the type of wheat, the parts of wheat included, the processing of the wheat. Wheat flour is used daily in most of the states in north as well as western states of India. Wheat flour is used to make rotis, parathas etc for daily meal. There are various other uses such as in bread and other bakery products as well as in many other recipes in which wheat flour is used as main ingredient.

Literature Review:

Simulation modeling, artificial intelligence, and optimization techniques have become indispensable tools in engineering and construction management, particularly in understanding complex systems and improving human-machine interaction. The following literature highlights significant contributions in this domain.

Kannan Swaminathan, Michael G. Madden, and Gunnar Lucko (2009) emphasized the rapid deployment of simulation models in building construction applications. Their work, presented at the Winter Simulation Conference, proposed a modular approach that facilitates fast and effective model development tailored to the dynamic nature of construction projects. The methodology enables engineers to test alternatives and optimize construction operations without excessive computational effort.

J. Mason, R. R. Hill, L. Mönch, and O. Rose (1998) explored simulation optimization using mathematical programming. Their research focused on representing discrete event systems within optimization frameworks, allowing for better scheduling, resource allocation, and performance assessment in complex environments such as manufacturing or logistics systems. Their study illustrates how simulation can be seamlessly integrated with mathematical models to derive efficient solutions.

Yanxin Zhang (2005), in his Ph.D. dissertation, presented a 3D simulation model of manual material handling tasks using nonlinear optimization techniques. His research provided valuable insights into human biomechanical performance, fatigue analysis, and workplace design. The integration of 3D visualization and optimization helped in understanding ergonomic limitations and improving the safety of manual labor tasks.

F. Hosseinpour and H. Hajihosseini (2009) discussed the importance of simulation in the manufacturing sector, asserting that simulation provides a risk-free environment for testing production strategies. Published in the World Academy of Science, Engineering and Technology, their work advocates for the widespread adoption of simulation as a decision-support tool in modern manufacturing systems.

J. P. Modak (2009) proposed the application of artificial intelligence (AI) techniques for enhancing the performance quality of man-machine systems. His philosophical approach highlighted the synergy between AI models and physical systems to optimize productivity and reduce human effort. The study bridges theoretical AI principles with practical system improvements in industrial settings.

Flood I. and Christophilos P. (2009) contributed to the domain by modeling construction processes using artificial neural networks (ANNs). Their research, presented at the Oxford Business & Economics Conference, demonstrated how ANNs could learn from past data and predict construction outcomes, making them valuable tools for project planning and risk mitigation.

Finally, Zakiuddin K. S. and J. P. Modak (2010) formulated a data-based ANN model for a human-powered fodder chopper. Their study, published in the *Journal of Theoretical and Applied Information Technology*, provided a unique example of how neural networks can be applied to human-powered systems. The model helped in predicting performance metrics and optimizing human energy utilization in rural technologies.

Objectives of the Study:

- To derive relevant dimensionless Pi terms from a selected mathematical or physical model using the Buckingham Pi theorem.

- To classify the Pi terms into positive (πa), negative (πb), and dependent (π_8) categories based on their mathematical formulation and influence in the system.
- To develop a three-dimensional surface plot that illustrates the interaction between πa, πb, and π_8.

Hypothesis:

- **H_0 (Null Hypothesis):** There is no significant relationship between the combined Pi terms πa (positive product) and πb (negative product) and the dependent Pi term π_8 representing Human Energy Expenditure.
- **H_1 (Alternative Hypothesis):** There exists a statistically significant nonlinear relationship between πa, πb, and the dependent Pi term π_8. This relationship can be effectively represented through a three-dimensional surface plot and modeled using curve fitting and regression techniques.

Research Methodology:

1. Formulation of Pi Terms:

Using Buckingham's Pi theorem, seven dimensionless independent Pi terms were generated. These terms were categorized as:

- Positive Pi Terms: $\pi_1, \pi_2, \pi_4, \pi_5, \pi_7$
- Negative Pi Terms: π_3, π_6
- Dependent Pi Term: π_8 (representing Human Energy Expenditure)

Two compound Pi terms were defined as:

- $\pi a = \pi_1 * \pi_2 * \pi_4 * \pi_5 * \pi_7$
- $\pi b = \pi_3 * \pi_6$

2. Mathematical Modeling:

A generalized mathematical model was hypothesized in exponential form as:

$$\pi 8 = a0 \times \pi a^{a1} \times \pi b^{a2}$$

Taking logarithms on both sides:

$$\log(\pi 8) = \log(a0) + a1 \log(\pi a) + a2 \log(\pi b)$$

Let:

- $Z_1 = \log(\pi_8)$
- $A_1 = \log(\pi a)$
- $B_1 = \log(\pi b)$
- $K_1 = \log(a_0)$

Then, the regression equation becomes:

$$Z_1 = K_1 + a_1 A_1 + a_2 B_1$$

3. Data Collection and Curve Fitting:

- Observational data from experimental setups were used to compute values of the Pi terms.
- MATLAB's curve fitting toolbox was used to perform polynomial regression (Poly5 model).
- The least squares method was applied to find best-fit models between dependent and independent Pi terms.

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ISSN: 3068-272X www.ijdim.com Original Research Paper

4. Surface Plot Representation:

- Figure 1: Shows a 3D surface plot of πa, πb, and π_8, illustrating their mutual interaction.
- Figure 2: Displays a 2D plot of all points on the OX–OZ plane (πa vs π_8).
- Figure 3: Displays a 2D plot of all points on the OY–OZ plane (πb vs π_8).

Each plot helps visualize and verify the nonlinear behavior of the dependent Pi term.

5. Regression Model Results:

From matrix transformation and solving equations via MATLAB, the following coefficients were derived:

- $a_0 = 0.3395$
- $a_1 = 0.0087$
- $a_2 = -0.0187$

Final model equation:

$$\pi 8(Hr) = 0.3395 \times \pi a^{0.0087} \times \pi b^{-0.0187}$$

This model confirms the dependency of Human Energy Expenditure on the composite positive and negative Pi terms.

6. Goodness of Fit (Poly5 Curve Model):

Figure 2 (OX– OZ Axis):

- $R^2 = 0.237$ (Low predictability)
- RMSE = 0.00926

Figure 3 (OY– OZ Axis):

- $R^2 = 0.195$ (Very low predictability)
- RMSE = 0.00951

While individual 2D projections show weak correlation, the 3D surface model improves interpretability by capturing multi-term interactions.

Figure 1: 3-D Surface plot of πa (product of positive Pi terms) , πb (product of negative Pi terms) and dependent Pi term π_8.

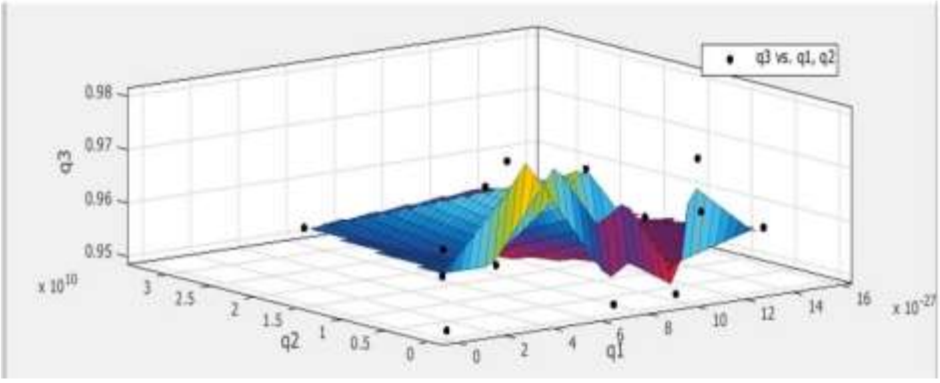
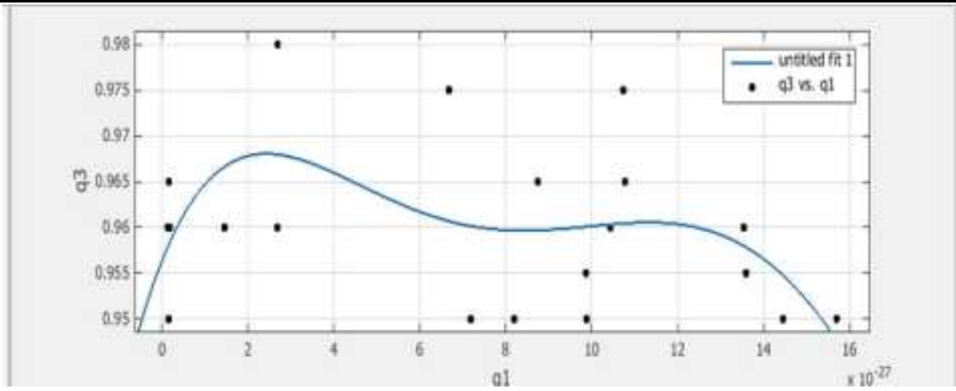


Figure 2:2-D Plot of all points on the OX OZ axis for dependent Pi term Oq



Linear model Poly5 is formulated using curve fitting tool to get the regression equation using least square method.

Linear model Poly5:

$$f(x) = p1 * x5 + p2 * x4 + p3 * x3 + p4 * x2 + p5 * x + p6$$

(1)

Coefficients (with 95% confidence bounds):

$$p1 = 4.373e + 128 \, (-3.099e + 129, 3.974e + 129)$$
$$p2 = -2.486e + 103 \, (-1.65e + 104, 1.153e + 104)$$
$$p3 = 4.809e + 77 \, (-1.484e + 78, 2.446e + 78)$$
$$p4 = -3.903e + 51 \, (-1.531e + 52, 7.5e + 51)$$
$$p5 = 1.18e + 25 \, (-1.228e + 25, 3.588e + 25)$$
$$p6 = 0.9564 \, (0.9445, 0.9682)$$

Goodness of fit:

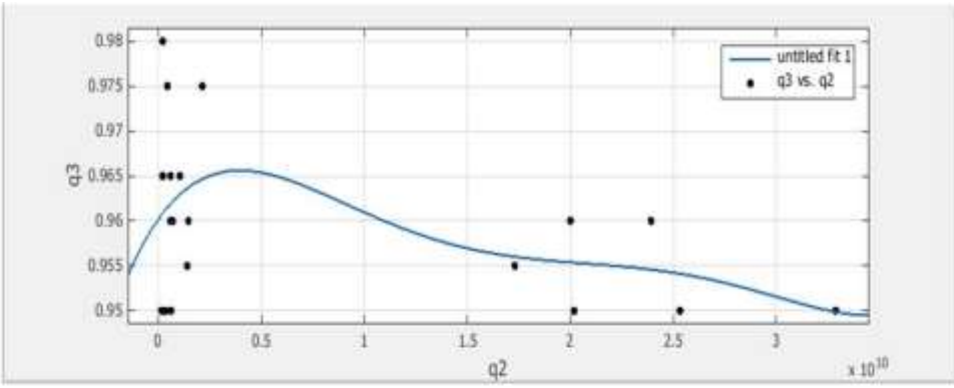
SSE: 0.001201

R-square: 0.237

Adjusted R-square: -0.03545

RMSE: 0.009261

Figure 3: 2-D Plot of all points on the OY OZ axis for dependent Pi term Oq



Linear model Poly5 is formulated using curve fitting tool to get the regression equation using least square method.

Linear model Poly5:

$$f(x) = p1 * x5 + p2 * x4 + p3 * x3 + p4 * x2 + p5 * x + p6$$

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(2)
Coefficients (with 95% confidence bounds):
$p1 = 1.081e-53\ (-3.022e-52, 3.238e-52)$
$p2 = -1.077e-42\ (-2.561e-41, 2.346e-41)$
$p3 = 3.897e-32\ (-6.077e-31, 6.856e-31)$
$p4 = -6.089e-22\ (-6.922e-21, 5.704e-21)$
$p5 = 3.239e-12\ (-1.407e-11, 2.055e-11)$
$p6 = 0.9602\ (0.9493, 0.971)$
Goodness of fit:
SSE: 0.001267
R-square: 0.1947
Adjusted R-square: -0.09285
RMSE: 0.009514

Formulation of Models Based on combination of observed data for dependent pi term Human Energy Expenditure

Model for dependent variable: Human Energy Expenditure

$\pi 8(Hr) = 0.1282\ \pi_1^{0.7394} * \pi_2^{0.0245} * \pi_3^{-0.1778} * \pi_4^{0.0765} * \pi_5^{0.0422} * \pi_6^{-0.0696} * \pi_7^{1.4237}$ (3)

Two more independent pi terms (π_a, π_b) were formed and already formed one dependent pi term (π_8) were decided during experimentation and hence are available for the model formulation.

π_a is formed by the product of the positive independent π as specified in equation (4).

$$\pi a = (\pi 1 * \pi 2 * \pi 4 * \pi 5 * \pi 7)$$

(5)

πb is formed by the product of the negative independent π as specified in equation (6).

$$\pi b = (\pi 3 * \pi 6)$$

(7)

Each dependent π term is the function of the available independent terms,

$$\pi 8(Hr) = f(\pi a, \pi b)$$

(8)

A probable exact mathematical form for the dimensional equations of the phenomenon could be relationships assumed to be of exponential form [26]. For example, the model representing the behavior of dependent pi term π_8 with respect to various independent pi terms can be obtained as under.

$\pi_8(Hr) = a_0 \quad \pi_a^{a1} * \quad \pi_b^{a2}$

(9)

Therefore two unknown terms in the equation 10 i.e. constant of proportionality a_0& indices a_1, a_2.

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The values of exponent are a_1 and a_2 are established independently at a time, on the basis of data collected through classical experimentation. There are three unknown terms in the equation (9) curve fitting constant a_0 and indices a_1 and a_2. To get the values of these unknowns we need minimum a set of three set of all unknown dimensionless pi terms.

$Z=A+b_X+C_Y$……

(10)

The equation (10) can be brought in the form of equation (11) by taking log on both sides.

Model of dependent pi term π_8 for process time

$\pi_8(Hr)=a_0\ \pi^{a1} * \pi_b^{a2}$

Taking log on the both sides of equation for π_8

$LOG\ \pi_8=\mathrm{LOG}\ a_0 + a_1\ \mathrm{LOG}\ \pi_a + a_2\mathrm{LOG}\ \pi_b$

(11)

Let, $Z_1 = \log \pi_8$, $K_1 = \log\ a_0$, $A_1 = \log \pi_a$, ,$B_1 = \log \pi_b$

Putting the values in equations 4, the same can be written as

$Z_1=K_1+ a_1*A_1+a_2*B_1$

(12)

Equation (12) is a regression equation of Z on A, B, C,D, E, F and G in a dimensional coordinate system

$\sum Z_1= n* K_1 +a_1\sum A_1+a_2\sum B_1$

$\sum Z_1A_1=K_1 \sum A_1 +a_1\sum A_1*A_1 +a_2\sum B_1*A_1$

$\sum Z_1B_1= K_1\sum B_1 +a_1\sum A_1*B_1+a_2\sum B_1*B_1$

(13)

In the above set of equations the values of the multipliers A1,B1 are substituted to compute the a_0, a_1 and a_2 in the set of equations.

After substituting these values in the equations (13) one will get a set of seven equations, which are mutinously to get the values of a_0, a_1 and a_2. The above equations can be verified in the matrix form and further values of a_0, a_1 and a_2 can be obtained by using matrix analysis.

$X_1=inv(W)*P_1$

Solving these equations using ‘ MATLAB’ is given below. W = 3 x 3 matrix multipliers of a_0, a_1 and a_2

P_1 = 3 x 1 matrix of the terms on L H S and X_1 = 3 x 1 matrix of values of a_0, a_1 and a_2

For deduction of combined independent Pi term π_a and π_b for Response variable Human Energy Expenditure from Observation Table refer Annexure 50

For Deduction of Combined independent Log Pi term π_a and π_b for Response variable Human Energy Expenditure from Observation Table refer Annexure 51

For calculation of matrix formation using Log Values of independent and dependent Pi

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ISSN: 3068-272X www.ijdim.com Original Research Paper

values (Human Energy Expenditure) refer Annexure 52

Matrix Transformation and deduction of indices for model Formulation for Human Energy Expenditure is depicted in Annexure 53

Solving Matrix Transformation and deduction of indices for model Formulation for Productivity using METLAB is depicted in Annexure 54

After solving we get

a_0 =0.3395, a_1 =0.0087, a_2 =-0.0187

Hence the model is

$$\pi_8(Hr) = 0.3395\, \pi_a^{\,0.0087} * \pi_b^{\,-0.0187} \qquad (12)$$

Conclusions Overall Results:

This study effectively demonstrates the utility of dimensional analysis and curve fitting in understanding complex relationships among multiple dimensionless variables in a physical system. Using the Buckingham Pi theorem, seven independent Pi terms were generated, from which two key independent Pi terms πa (positive product) and πb (negative product) and one dependent Pi term, π_8 (representing Human Energy Expenditure), were formulated.

Through the development of a 3D surface plot (Figure 1), the study visualizes the interaction among πa, πb, and π_8. The regression analysis using a Poly5 curve fitting model yielded a predictive equation:

$$\pi 8(Hr) = 0.3395 \times \pi a^{0.0087} \times \pi \mathrm{b}^{-0.0187}$$

This result indicates a mild positive sensitivity to πa and a weak negative sensitivity to πb, signifying that increases in positively contributing Pi terms have a slightly amplifying effect on Human Energy Expenditure, while negative contributors reduce it marginally.

Although the R-square values for the 2D regression models (Figures 2 and 3) were relatively low (R^2 = 0.237 and R^2 = 0.195 respectively), the overall surface model provided a better qualitative understanding of how these variables interact in three dimensions.

The matrix-based computation and use of MATLAB provided a robust methodological framework for determining the constants of proportionality and exponents, validating the model with actual observational data.

Future Scope of the study:

- Improved Model Accuracy:
  Future work can focus on collecting a larger and more diverse dataset to enhance the regression model's accuracy and increase the R^2 value, leading to better predictive capability.
- Multi-variable Optimization:
  This modeling approach can be extended to include more Pi terms or multivariate combinations to study energy expenditure or other dependent phenomena in detail.
- Application in Biomedical and Sports Sciences:
  The model for Human Energy Expenditure could be refined and applied in fields such as ergonomics, exercise physiology, and human performance analysis, especially in designing equipment, workspaces, or workout plans.
- Integration with Machine Learning Models:
  Combining dimensional analysis with ANNs, SVMs, or other ML algorithms may lead to

hybrid models with improved accuracy, adaptability, and generalization to unseen data.

- 3D and 4D Visual Analytics Tools: More advanced tools can be developed to visualize not just static 3D plots, but time-varying or interactive 4D surfaces, for enhanced real-time decision-making.
- Extension to Other Physical Phenomena: The approach can be adapted for modeling phenomena like fluid flow, heat transfer, material strength, or pollution dispersion where dimensionless analysis is beneficial.

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